A lineac-mass spectrometer by induced Hall potential for electromagnetic isotopic separation working at high pressures

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In this paper a novel alternative for bulk electromagnetic separation working at high pressures is proposed. It is shown that if a self-induced Hall potential is stimulated in the boundaries, the system will be able to take advantage of the collisions process boosting the isotopic separation and resulting in a linear-spectrometer with a higher spatial separation per length than a traditional calutron. Although originally the concept has been devised for obtention of medical radioisotopes where the minorities isotopes desired to be separated come from neutron capture and the is the heavier isotope, however, if the Hall potential is substituted by an external electrical field, the concept is equally applicable for species where the minorities isotope is the lighter as for example the enrichment of uranium. Additional R&D is required to explore the possibilities of this concept as well as the optimization of several variables.

Keywords. *electromagnetic separation, Calutron*

I. INTRODUCTION

Since the late 1940s the dominant source of most of the enriched stable isotopes produced in the United States has been the calutrons. These large separators were originally developed to separate the isotopes of uranium in World War II. The technology that they employ, which has been relatively unchanged for the last 50 years, is based on the electromagnetic separation of elemental material into its constituent,[1].

Nevertheless, despite the robustness and simplicity of calutrons they are very inefficient in terms of processed material but very efficient in terms of product purity. For example, the plasma separation process which has been developed in the last years at the Oak Ridge National Laboratory (ORNL) is only about half as efficient as the process performed by the calutrons in terms of product purity, but because a calutron only works at very low pressures (a millionth of an atmosphere) then the plasma separation process is 300 times faster, [3].

In an attempt to solve the very low throughput of calutrons, the department of engineering at the University of Cambridge is developing an alternative approach which permits working with higher concentration of material, i.e., higher pressure, but at the same time maintaining the robustness and reliability of a calutron, additionally it will be also demonstrated that the spatial-power separation of the proposed concept is at least 5 times higher than a classical calutron . which translates in a smaller dimensions than traditional calutrons.

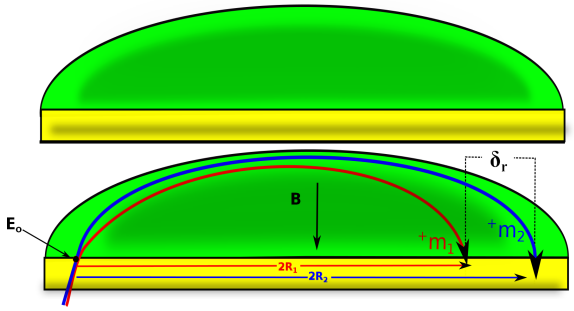

FIG. 1: Sketch of a classical calutron mass spectrograph.

The idea proposed is resulting in a linear spectrograph boosted by an induced Hall potential. In this first paper, the theoretical statement behind of this concept is for first time outlined.

II. THE ELECTROMAGNETIC ISOTOPIC SEPARATION BOOSTED BY INDUCED HALL POTENTIAL

A. statement of concept

Fig. 1 is a sketch of a classical calutron and Fig. 2 and Fig. 3 the proposed linear mass spectrometer which will be explained below.

For the sake of illustration, let assume only to isotopes

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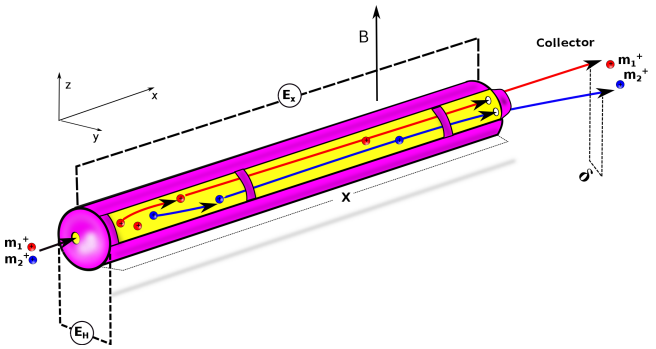

FIG. 2: The proposed linear mass spectrometer for bulk-isotopic electromagnetic separation

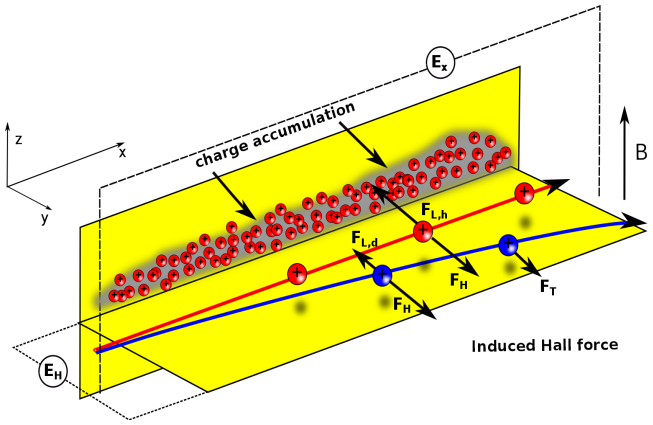

FIG. 3: The Lorentz force on the lighter and more abundant isotope m_1 is exactly compensated by the hall force, however this is not the case for the heavier isotope m_2.

to be separated, and also let assumes that the majority isotope has a mass m_1 and the minority isotope a mass m_2 being $m_1 < m_1$. This assumption is certainly the most typical case for most of the radioisotopes with significant importance in medicinal, for example, the important radioisotope ^{99}Mo which comes from the neutronic irradiation of stable molybdenum. In general, this assumption is accomplished for most of the radioisotopes which comes from neutronic irradiation as

$$A + n \rightarrow B \tag{1}$$

where the isotope A is the element irradiated, say, in a nuclear reactor, and the isotope B is the desired isotope which is gaining one or more neutrons, and then the condition $m_1 < m_2$ is satisfied. However, even if the above condition is not accomplished, for example, for uranium enrichment where the desired isotope ^{235}U is lighter than the bulk isotope ^{238}U, it is still possible apply the proposed concept, see Appendix.

First of all, referring to Fig 2 and 3, before we apply the electrical field E_x the ions of the heavier isotope m_2 and lighter isotope m_1 as well as the neutral atoms (no ionized) are moving randomly owing to its thermal kinetic energy, so, the average velocity is zero. Under the effect if the longitudinal electric field E_x, ions will move in the direction of the applied force (See Fig. 3) with a drift velocity Δv_x which is in general much slower than the mean thermal velocity. The drift velocity is the maximum velocity attained by the between collisions, (See Fig. 3). taking into account that the longitudinal acceleration acting on the ion i, is given by

$$a_{x,i} = \frac{q\mathbf{E}_x}{m_i} \tag{2}$$

then the drift velocity of the ions i is approximately given by

$$\Delta v_{x,i} = \frac{q\mathbf{E}_x}{m_i}\tau_i \tag{3}$$

where τ_i is the mean free time between collisions. According to kinetic theory, the mean free time τ_i is related by the scatering cross section as

$$\tau_i \cong \frac{1}{N_g \sigma v_{x,i}} \tag{4}$$

where

$$\frac{m_i v_{x,i}^2}{2} = \bar{e} = \frac{3}{2}\kappa T \tag{5}$$

is the mean thermal kinetic energy being κ the Boltzmann constant and T the mean temperature of the gas.

Now, in the presence of the magnetic field $\mathbf{B}$, there will be a time-averaged magnetic force given by the Lorentz force equation,

$$F_{i,y}^L = q\Delta v_{x,i}\mathbf{B}_z \tag{6}$$

and inserting the drift velocity expression in Eq.(4) yields

$$F_{i,y}^L = -\frac{q^2 E_x \tau_i \mathbf{B}_z}{m_i} \tag{7}$$

Now, because the boundary confinement conditions, an accumulation of charges will result so that the net current in the $y-$ direction is zero.As a result of this boundary condition, an electric field develops -called as Hall potential- that balances the magnetic force on the drifting ions. To maintain the condition of zero current flow in the $y-$ direction, the induced Hall force is given by the same Eq.(7) but with opposed sign, i.e,

$$F_{i,y}^H = \frac{q^2 E_x \tau_i \mathbf{B}_z}{m_i} \tag{8}$$

However, the accumulation of charge in the confinement boundary is given by the accumulation of majority carriers which in our case will be the ions of mass m_1. Thus Eq.(8) becomes

$$F_y^H = \frac{q^2 E_x \tau_1 \mathbf{B}_z}{m_1} \tag{9}$$

which is acting on the ions of 1 with mass m_1 and ions 2 with mass m_2, indistinctively, i.e.,

$$F_y^H = F_{y,1}^H = F_{y,2}^H \tag{10}$$

Thus, the net force on the m_1 ions will be zero because the counterbalancing force is created by the accumulation of these ions. But, however, because the Lorentz force acting on the heavier ions m_2 is smaller than the Lorentz force because its lower thermal velocity in comparison with the ions m_1 for an equal thermal energy, see Eq. (5), then a net force on ions m_2 is developed which propels them in the opposed direction of the Lorentz force.
Thus , the net force of motion acting on the ions of mass m_2 is given by

$$F_{y,2}^m = F_y^H + F_{y,2}^L \tag{11}$$

or taking into account Eq.(9), Eq.(7), Eq.(5) and Eq.(4) becomes as

$$F_{y,2}^m = \frac{q^2 E_x \mathbf{B}_z}{N_g \sigma \sqrt{2\bar{e}}} \left[\frac{1}{\sqrt{m_1}} - \frac{1}{\sqrt{m_2}} \right] \tag{12}$$

Finally, the Hall displacement δ_y may be calculated by the following dynamic Equation,

$$\delta_y == \frac{1}{2} a_{y,2} t_2^2 \tag{13}$$

where $a_{y,2}$ is the total acceleration in the y-direction of ions of mass m_2, i.e, $a_{y,2} = \frac{F_{y,2}^m}{m_2}$ and t_2 is the total flight-time that ions of mass m_2 need to pass through the chamber. If the length of the chamber is X, then the flight time is given by

$$t_2 = \frac{X}{\Delta v_{x,2}} \tag{14}$$

and then Eq.(13) becomes

$$\delta_y = \frac{1}{2} \frac{F_{y,2}^m}{m_2} \frac{X^2}{\Delta v_{x,2}^2} \tag{15}$$

or

$$\delta_y = 2\sqrt{\bar{e}} N_g \sigma \frac{\mathbf{B}_z}{\mathbf{E}_x} \left[\frac{1}{\sqrt{m_1}} - \frac{1}{\sqrt{m_2}} \right] \cdot X^2 \tag{16}$$

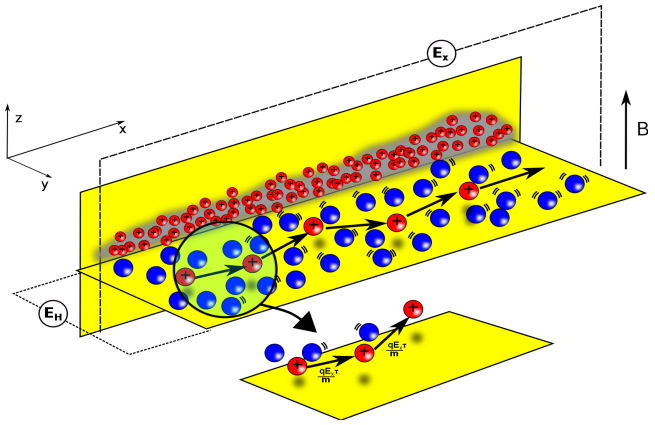

FIG. 4: As a result of the collisions process and the external longitudinal electrical field applied, a drift velocity, Δv_x is developed.

B. discussion

It is interesting to see that under the assumptions made, the separative power of the proposed linear-spectrometer increases inasmuch that the concentration of gas (or pressure) in the chamber increases, and then it is eliminating the limitations of a calutron which only works at very low pressures resulting in a very low throughput. In this way the proposed spectrometer boosted by self-induced Hall potential appears as an attractive alternative for process where industrial isotopic separation is needed.

Although, a classical calutron as depicted in Fig. 1 can not working at high pressures as was previously mentioned, nevertheless it is interesting to compare the separative power from the proposed linear spectrometer and the calutron.

If it is plausible to define a separative power as the separative distance between isotopes at the collector, δ per length of path as

$$\Gamma = \frac{\delta}{l} \tag{17}$$

then the separative power for our lineartron yields

$$\Gamma_{lin} = 2\sqrt{\bar{e}} N_g \sigma \frac{\mathbf{B}_z}{\mathbf{E}_x} \left[\frac{1}{\sqrt{m_1}} - \frac{1}{\sqrt{m_2}} \right] \cdot X \tag{18}$$

On the other hand, the separative power for a calutron as defined in Eq/(19) and depicted in Fig. 1 will be

$$\Gamma_{cal} = \frac{\delta_r}{\pi R_i} \tag{19}$$

where R_i is the gyro-radius of the i ion given by

$$R_i = \frac{\sqrt{2\bar{e}}}{q\mathbf{B}_z}\sqrt{m_i} \tag{20}$$

and taking into account that $\delta_r = 2(R_2 - R_1)$, and taking the maximum radius in Eq.(19), i.e; $R_i = R_2$, then the separative power yields

$$\Gamma_{cal} = \frac{2}{\pi}\left[1 - \sqrt{\frac{m_1}{m_2}}\right] \tag{21}$$

And the separative power from our lineatron can be expressed as function of the separative power of the calutron as

$$\frac{\Gamma_{lin}}{\Gamma_{cal}} = \pi\frac{\sqrt{\bar{e}}}{\sqrt{m_1}}N_g\sigma\frac{\mathbf{B}_z}{\mathbf{E}_x}\cdot X \tag{22}$$

In order to do an straightforward comparison, let assume that the dimensions of the cambutron X is the same order than the calutron, i.e $X \approx 2R_2$, then inserting Eq.(20) into eq.(22) one obtains,

$$\frac{\Gamma_{lin}}{\Gamma_{cal}} = 2\pi\sqrt{2}\cdot\frac{\bar{e}N_g\sigma}{E_x q}\sqrt{\frac{m_2}{m_1}} \tag{23}$$

To obtain some idea of the ration between separative power predicted by Eq.(23), we assume some typical values of the parameters: $\frac{m_2}{m_1} = 1.01$, for example for the system $^{99}Mo/^{98}Mo$; $N_g\sigma \sim 0.02cm^{-1}$; $\bar{e} \sim 1.0keV$, $E_x = 50V/cm$; and then $\frac{\Gamma_{lin}}{\Gamma_{cal}} \approx 3$. This result can be interpreted in two different ways namely: on one hand the lineatron will have about 3 times more spatial-separation between ions when they arrive at the collector than a calutron with the same size, or on the other hand, a lineatron will be five times smaller than a calutron and featuring the same spatial separation.

III. APPENDIX

It is possible that the minority isotope is not the heavier but the lighter, for example in the important ins=ndustrial process for uranium enrichment where is the ^{235}U isotope is lighter that the ^{238}U but is also the less abundant.
In this situation, the Hall potential given in Eq.(9)is not produced by the accumulation of ions m_1 but because the accumulation of ions m_2 which are the more abundant.

However, the Lorentz force acting on the lighter isotopes m_1 is higher than the Lorentz force acting on the heavier isotopes m_2, and then the induced Hall potential generated by the accumulation of m_2 charges will be unable to neutralize the Lorentz force acting on isotopes m_1 and as result they will precipitate at the wall.

However, in such situations, is enough applying an external electrical field E_y with the same potential defined by Eq. (9).

IV. CONCLUSIONS

In this manuscript an alternative device for isotopic electromagnetic separation working at high pressures was proposed and discussed. It is shown that promoting a self-induced Hall potential it is possible to obtain a linear-mass-spectrometer (lineatron) more efficient than a traditional calutron in terms of spatial separation with similar size, and working at higher pressures resulting in higher throughput and then the production of relevant quantities.
Additional R&D is required to explore the possibilities of this concept as well as the optimization of several variables.

NOMENCLATURE

a = acceleration
$\mathbf{B}_z$ = magnetic field z-direction
$\bar{e}$ = thermal kinetic energy
$\mathbf{E}$ = electrical field
F = force
F^H = Hall force
F^L = Lorentz force
m = mass
N_g = concentration atoms gas
q = charge ion-isotope
t = flight-time
T = temperature
v = velocity
X = longitudinal length of chamber

Greek symbols

τ = the mean free time between collisons
σ = scattering microscopic cross section

Subscripts

cal = calutron
lin = lineatron
1 = the lighter and more abundant isotope
2 = the heavier and less abundant isotope
i = ion
x = x-direction
y = y-direction
z = z-direction

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